Supporting Rigorous Mathematics Teaching and Learning

Identifying Strategies for Modifying Tasks to Increase the Cognitive Demand

PARTICIPANT HANDOUT

Tennessee Department of Education

High School Mathematics

Rationale

There is no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics.

Lappan & Briars, 1995

By determining the cognitive demand of tasks and being cognizant of those features of tasks that contribute to their cognitive demand, teachers will be able to create opportunities for students to engage in rigorous mathematics learning.

Session Goals

Participants will:

- deepen understanding of the cognitive demand of a task;
- learn strategies for increasing the cognitive demand of a task; and
- recognize how increasing the cognitive demand of a task gives students access to the Common Core State Standards (CCSS) for Mathematical Practice.

Overview of Activities

Participants will:

- discuss and compare the cognitive demand of mathematical tasks;
- identify strategies for modifying tasks to increase their cognitive demand; and
- modify tasks to increase their cognitive demand.

Comparing the Cognitive Demands of Tasks

Take 5 minutes to individually examine the tasks in this handout. You will find it helpful to think carefully about how students might answer each of the questions.

Work together as a team to identify the cognitive demands of each task using the Mathematical Task Analysis Guide (TAG).

What is it that makes a high-level task "high level"?

Be prepared to present and justify your conclusions to the whole group. Be sure to identify the mathematical understandings students will have an opportunity to grapple with as they work to solve the task(s).

Comparing the Cognitive Demands of Tasks (cont.)

Compare the original versions of the tasks with the modified versions.

- How are the modified tasks the same and how are they different from the original?

- In what ways was the original task modified, and for what purpose?

- What is the "value added" by making the modification to the original task?

  - Which CCSS for Mathematical Practice will students use when solving each task?

  - Which CCSS for Mathematical Content are the focus of each task?

The Mathematical Task Analysis Guide

Lower-Level Demands	Higher-Level Demands
Memorization Tasks • Involve either producing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	Procedures With Connections Tasks • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations, or explanations that focus solely on describing the procedure that was used.	Doing Mathematics Tasks • Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships. • Demands self-monitoring or self-regulation of one's own cognitive processes. • Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

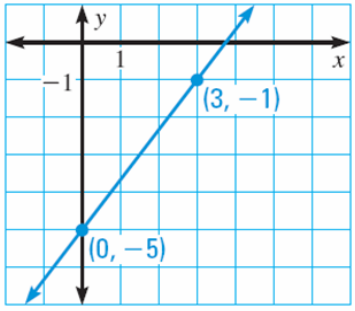
Stein, Smith, Henningsen, & Silver (2000). Implementing standards-based mathematics instruction: A casebook for professional development, p. 16. New York: Teachers College Press.

Equations and Graphs Task
Write an Equation

Original

EXAMPLE 3 Write an equation of a line given two points

Write an equation of the line shown.

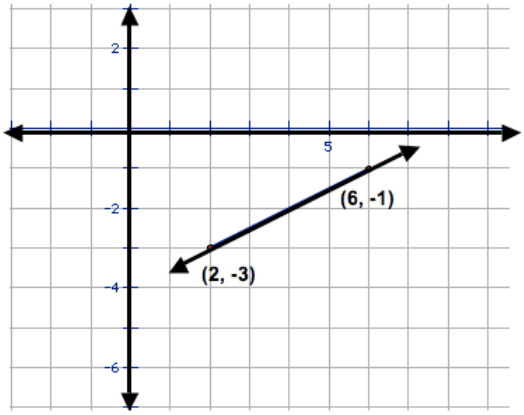


from Larson, R., Boswell, L., Kanold, T. D., & Stiff, L. (2007). Algebra 1, McDougal Littell, p. 284

Modification

Example 3. Write an equation of a line given 2 points.

a. Write an equation of the line shown.



b. Explain how you can write the equation of any line if you know 2 points on the line.

Slope Intercepts

Rewrite the equation in slope-intercept form. Identify the slope and y-intercept.

1. $y - 4 = 3x$
2. $x = -y + 2$
3. $2x + y = 6$
4. $5x + 8y = 32$
5. $4x - 3y = 24$
6. $-27 + 9y + 18 = 0$

Algebra 1: Concepts and Skills, 2001, p. 258. McDougal Littell

Modification

Think of a real-life situation that can be modeled by each of the following equations:

$y = -2x + 6$

$y = 2x + 6$

Explain the meaning of the constant and coefficient in terms of each problem.

Slope Task

Original

Find the slopes of the lines containing these points.

11. $(4, 0)$ $(5, 7)$

12. $(3, 0)$ $(6, 2)$

13. $(0, 8)$ $(-3, 10)$

14. $(0, 9)$ $(4, 7)$

15. $(3, -2)$ $(5, -6)$

16. $(-2, 4)$ $(6, -7)$

17. $(0, 0)$ $(-3, -9)$

18. $(0, 0)$ $(-4, -8)$

19. $\left(\frac{3}{4}, \frac{1}{2}\right)$ $\left(\frac{1}{4}, -\frac{1}{2}\right)$

20. $\left(\frac{1}{4}, \frac{1}{8}\right)$ $\left(\frac{1}{2}, \frac{3}{4}\right)$

Smith, S., Charles, R., Dossey, J., & Bittinger, M. (2001). Algebra 1, California Edition, Prentice Hall, p. 321

Modification

For each pair of points, find a third point that is on the same line. Explain how you used the information given to find the third point.

11. $(4, 0)$ $(5, 7)$

12. $(3, 0)$ $(6, 2)$

13. $(0, 8)$ $(-3, 10)$

14. $(0, 9)$ $(4, 7)$

Electricity Rates

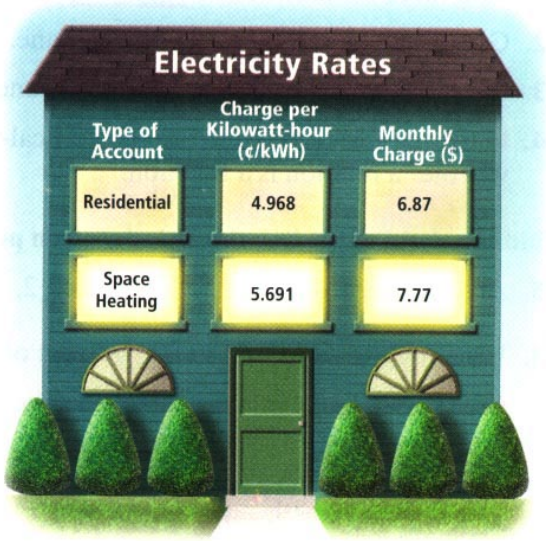
Original

Data Analysis **Use the data at the right for Exercises 22–24.**

22. Write a function rule for the total monthly charge to a customer who has electric space heating. (*Hint:* Remember to write the rule either in dollars or in cents.)

23. **a.** Write a function rule for the total monthly charge to a residential customer.
    b. Suppose a residential customer received a monthly bill for $22.52. How many kilowatt-hours did the customer use that month?

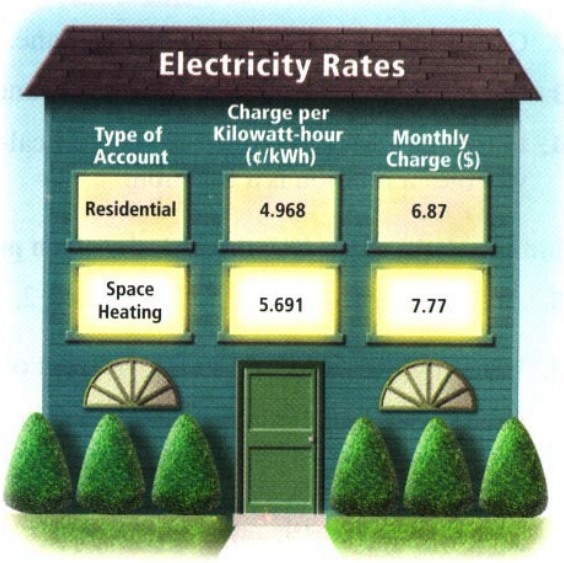
24. **Critical Thinking** Write a rule to describe how much a customer saves by being a residential customer rather than a customer who uses electricity for space heating.



Davison, D., Landau, M., McCracken, L., & Thompson, L. (2001). Pre–Algebra, California Edition, Prentice Hall, p. 407.

Modification

Write a function to describe how much a customer saves by being a residential customer rather than a customer who uses electricity for space heating.



Burning Calories While Swimming

Original

37. If you swim the backstroke, you burn 9 cal/min (calories per minute). If you swim the butterfly stroke, you burn 12 cal/min. The equation $9x + 12y = 360$ models how you can burn 360 cal by swimming the backstroke for x min and the butterfly for y min.
   a. Find the solutions of the equation for $x = 0$ and $y = 0$. Explain what your solutions mean.
   b. Graph the solutions you found in part (a). Draw a line through the two points.
   c. The solutions you found in part (a) are the *y-intercept* and the *x-intercept* of the graph. Explain why these names are appropriate.
   d. Use your graph from part (b). If you swim the butterfly stroke for 10 min, how long should you swim the backstroke to burn a total of 360 calories?

Davison, D., Landau, M., McCracken, L., & Thompson, L. (2001). Pre–Algebra, California Edition, Prentice Hall, p. 395.

Modification

37. If you swim the backstroke, you burn 9 cal/min (calories per minute). If you swim the butterfly stroke, you burn 12 cal/min.

   a. You want to burn 360 calories with your swimming workout. Determine a way to find all the combinations of swimming the butterfly and backstroke that meet this goal.

   b. Write and graph an equation that represents the relationship described in part a.

   c. Use your graph to answer the following:

      What happens when you only swim the backstroke?

      What happens when you only swim the butterfly?

   d. If you swim the butterfly stroke for 10 minutes, how long should you swim the backstroke to burn a total of 360 calories?

Parallel Lines Postulates

Original

ACTIVITY EXPLORE PARALLEL LINES

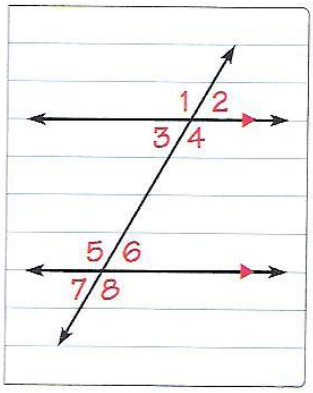
Materials: lined paper, tracing paper, straightedge

STEP 1 **Draw** a pair of parallel lines cut by a nonperpendicular transversal on lined paper. Label the angles as shown.

STEP 2 **Trace** your drawing onto tracing paper.

STEP 3 **Move** the tracing paper to position $\angle 1$ of the traced figure over $\angle 5$ of the original figure. Compare the angles. Are they congruent?

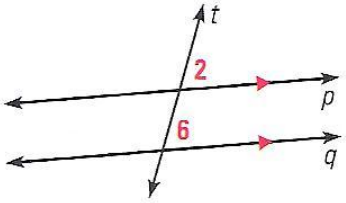
STEP 4 **Compare** the eight angles and list all the congruent pairs. What do you notice about the special angle pairs formed by the transversal?



POSTULATE For Your Notebook

POSTULATE 15 Corresponding Angles Postulate

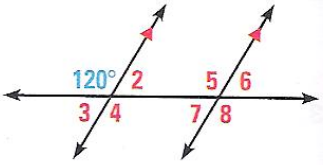
If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.



$\angle 2 \cong \angle 6$

EXAMPLE 1 Identify congruent angles

The measure of three of the numbered angles is 120°. Identify the angles. Explain your reasoning.



Solution

By the Corresponding Angles Postulate, $m\angle 5 = 120°$.
Using the Vertical Angles Congruence Theorem, $m\angle 4 = 120°$.
Because $\angle 4$ and $\angle 8$ are corresponding angles, by the Corresponding Angles Postulate, you know that $m\angle 8 = 120°$.

Larson, R., Boswell, L., Kanold, T. D., & Stiff, L. (2007). Geometry, McDougal Littell, p. 154

Parallel Lines Postulates (cont.)

Modification

Activity:	Explore Parallel Lines
Materials:	lined paper, tracing paper, straightedge
Step 1	***Draw*** a pair of parallel lines cut by a non-perpendicular transversal on lined paper.
Step 2	***Trace*** your drawing onto tracing paper.
Step 3	***Number*** each of the eight visible angles from 1 to 8.
Step 4	***Trace*** the angles at the top of your drawing. Use the tracings to determine which, if any, other angles are congruent to those in the tracing. What do you notice?
Step 5	***Repeat*** Steps 1 to 4 using a transversal with a different slope.

Compare your results with those of your teammates and make conjectures about the angles formed by two parallel lines cut by a transversal.

Predict and test what will happen if 3 or more parallel lines are cut by the same transversal.

Adapted from Larson, R., Boswell, L., Kanold, T. D., & Stiff, L. (2007). Geometry, McDougal Littell, p. 154

Increasing the Cognitive Demand of Mathematical Tasks

What strategies for increasing the cognitive demand of tasks may be generalized from the modifications we have just examined?

The Common Core State Standards (CCSS)

How does the modification of a task to be a high level of cognitive demand impact the opportunities to make use of the Mathematical Practice Standards?

Strategies for Modifying Textbook Tasks

Increasing the cognitive demand of tasks:

- Ask students to create real-world stories for "naked number" problems (e.g., Slope Intercepts Task).
- Include a prompt that asks students to represent the information another way (with a picture, in a table, a graph, an equation, with a context) and to write about what insights they can gain about the math concept from the new representation (e.g., Slope Task, Slope Intercepts Task).
- Solve an "algebrafied" version of the task (e.g., Write an Equation, Electricity Rates).
- Use a task "out of sequence" before students have memorized a rule or have practiced a procedure that can be routinely applied (e.g., Slope Task, Equations and Graphs Task).
- Eliminate components of the task that provide too much scaffolding (e.g., Electricity Rates, Explore Parallel Lines).
- Adapt a task so as to provide more opportunities for students to think and reason—let students figure things out for themselves (e.g., Equations and Graphs Task, Burning Calories While Swimming, Explore Parallel Lines).

Your Turn

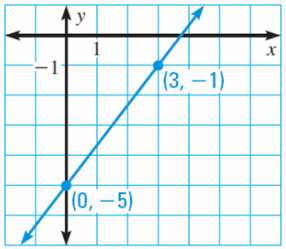
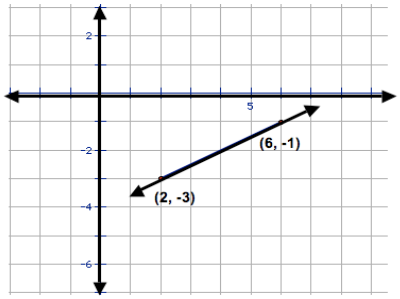
- Form grade-level groups of no more than three people.
- Briefly discuss important NEW mathematical concepts, processes, or relationships you will want students to uncover during the lesson.
- Examine your resources for a task that can (or will) give students a chance to engage in examining those concepts, processes, or relationships.
- Analyze the task and consider:
  - the CCSS for Mathematical Content; and
  - the CCSS for Mathematical Practice.
- Modify the textbook task by using one or more of the Textbook Modification Strategies.
- Be prepared to share your choices and your rationale.

Comparing Tasks

What messages to students do the differences in the tasks send?

Step Back

What have you learned about modifying tasks to increase the cognitive demand that you will use in your planning and instruction next school year?

Write an Equation	*Original Task*	*Modification*
	EXAMPLE 3 Write an equation of a line given two points Write an equation of the line shown.  from Larson, R., Boswell, L., Kanold, T. D., & Stiff, L. (2007). Algebra 1, McDougal Littell, p. 284	Example 3. Write an equation of a line given two points. 1. Write an equation of the line shown.  2. Explain how you can write the equation of any line if you know 2 points on the line.
Mathematical Point of the Exercise	Write an equation of a line given two points.	Write an equation of a line given two points then generalize this process.
Classification on TAG	Procedures Without Connections	Procedures With Connections
Reasons for Low-Level or High-Level Categorization	• Involves use of previously learned formula, since such problems usually appear as exercises following the introduction of the point/slope formula. • There is no connection between the concept or the meaning underlying the formula for finding the slope of two points. • There is little ambiguity about what needs to be done or how to do it. • Focus is on correct answer, not understanding or meaning.	• Although Part 1 requires the use of the slope-intercept form of a linear equation, students must struggle with a way to make sense of the y-intercept in the example. Generalizing in part 2 requires sense-making about what the formula means and determining ways to find the y-intercept when the graph does not pass through a readily readable point. • Part 2 requires the student to generalize the process they used in part 1. • Well-rehearsed procedures may be used to answer part 1, however some degree of cognitive effort is required in part 2 to generalize this process. • Part 2 focuses on the meaning of the variables in the formula for an equation of a line.

Promising Feature	Requires moving from a graphical to a symbolic algebraic representation.	Supports development/understanding of the point-slope form of a linear equation.
Modifications Made to the Task		The points are changed so that the y-intercept is not given, thus eliminating the possibility that the student may merely use the slope-intercept formula, $y = mx + b$. The additional prompt in part 2 asks the student to generalize the process they used in part 1.
Common Core State Standards		A.CED.A.2, A.CED.A.3, A.REI.D.10, MP1, MP2, MP3, MP4, MP6, MP7

Slope Intercepts	*Original Task*	*Modification*
	Rewrite the equation in slope-intercept form. Identify the slope and y-intercept. 1. y - 4 = 3x 2. x = -y + 2 3. 2x + y = 6 4. 5x + 8y = 32 5. 4x - 3y = 24 6. -27 + 9y + 18 = 0 Algebra 1: Concepts and Skills, 2001, p. 258. McDougal Littell	Think of a real-life situation that can be modeled by each of the following equations: y = -2x + 6 y = 2x + 6 Explain the meaning of the constant and coefficient in terms of each problem.
Mathematical Point of the Exercise	Practice re-writing equations into slope-intercept form. Recognize slope and intercept in slope-intercept form.	Recognize that an equation often models a real-world situation. Contextually recognize and explain the difference between a positive and negative slope, and explain the meaning of slope and intercept in a context.
Classification on TAG	Procedures Without Connections	Procedures With Connections
Reasons for Low-Level or High-Level Categorization	• Has a very weak connection to the concepts or meaning that underlie the procedure being used. • Is algorithmic. Use of a procedure is evident based on prior instruction, experience, or placement of the task. • Is focused on producing a correct answer rather than developing mathematical understanding. • Does not require self-regulation or organizing of thinking. • Requires no explanation.	• Suggests a pathway to follow (use of the slope-intercept form of a linear equation) that is a general procedure with close connections to the underlying concept (reasoning about the meaning of slope and intercept in a context). • Focuses student attention on contextual meaning of a mathematical model; on the difference between a positive and a negative slope in a model; and on the meaning of the intercept in a model. • Requires some degree of cognitive effort, since students must themselves build contexts where the intercept is the same, but the slopes are opposite.

Promising Feature	Recognize slope-intercept form of a linear equation	Recognize and make sense of slope-intercept form of a linear equation.
Modifications Made to the Task		Require students to recognize the meaning of slope and intercept from an equation in slope-intercept form, in a context of their own making. Require students to distinguish the impact of positive and negative slopes in context.
Common Core State Standards		A.SSE.A.1, A.SSE.A.1a, F.LE.A.1b, F.LE.B.5, MP1, MP2, MP3, MP6, MP7

MODIFYING COGNITIVE DEMAND OF TASKS MODULE -- ANSWER KEY

Slope Task	*Original Task*	*Modification*
	Find the slopes of the lines containing these points. **11.** $(4, 0)$ $(5, 7)$ **12.** $(3, 0)$ $(6, 2)$ **13.** $(0, 8)$ $(-3, 10)$ **14.** $(0, 9)$ $(4, 7)$ **15.** $(3, -2)$ $(5, -6)$ **16.** $(-2, 4)$ $(6, -7)$ **17.** $(0, 0)$ $(-3, -9)$ **18.** $(0, 0)$ $(-4, -8)$ **19.** $\left(\frac{3}{4}, \frac{1}{2}\right)$ $\left(\frac{1}{4}, -\frac{1}{2}\right)$ **20.** $\left(\frac{1}{4}, \frac{1}{8}\right)$ $\left(\frac{1}{2}, \frac{3}{4}\right)$ Smith, S., Charles, R., Dossey, J., & Bittinger, M. (2001). Algebra 1, California Edition, Prentice Hall, p. 321 .	For each pair of points, find a third point that is on the same line. Explain how you used the information given to find the third point. **11.** $(4, 0)$ $(5, 7)$ **12.** $(3, 0)$ $(6, 2)$ **13.** $(0, 8)$ $(-3, 10)$ **14.** $(0, 9)$ $(4, 7)$
Mathematical Point of the Exercise	Determine the numerical slope of a line given two points on the line.	Recognize the rate of change of a line given two points on the line and use the rate of change to name another point on the line.
Classification on TAG	Procedures Without Connections	Procedures With Connections
Reasons for Low-Level or High-Level Categorization	• Involves use of previously learned formula, since such problems usually appear as exercises following the introduction of the point/slope formula. • There is no connection between the concept or the meaning underlying the formula for finding the slope of two points. • There is little ambiguity about what needs to be done or how to do it. • Focus is on correct answer, not understanding or meaning.	• Focuses attention on finding and *using* the slope or rate of change to find another point, which requires understanding the meaning of the rate of change of the two variables. • Requires some degree of cognitive effort, since students must decide how to use the slope to find the third point. • May be represented in more than one way, either numerically or graphically.
Promising Feature	Use points on a line to find another attribute of the line.	Recognize and use the constant rate of change as a hallmark of a linear relationship.

Modifications Made to the Task		Asking for a third point moves the task from merely finding the slope to thinking about how to use the slope to find another point on the line.
Common Core State Standards		A.REI.D.10, F.IF.B.6, F.LE.A.1b, MP1, MP2, MP3, MP4, MP6. MP7, MP8

Electricity Rates	*Original Task*	*Modification*
	Data Analysis Use the data at the right for Exercises 22–24. 22. Write a function rule for the total monthly charge to a customer who has electric space heating. (*Hint:* Remember to write the rule either in dollars or in cents.) 23. a. Write a function rule for the total monthly charge to a residential customer. b. Suppose a residential customer received a monthly bill for $22.52. How many kilowatt-hours did the customer use that month? 24. **Critical Thinking** Write a rule to describe how much a customer saves by being a residential customer rather than a customer who uses electricity for space heating.  Davison, D., Landau, M., McCracken, L., & Thompson, L. (2001). Pre–Algebra, California Edition, Prentice Hall, 407.	 Write a function to describe how much a customer saves by being a residential customer rather than a customer who uses electricity for space heating
Mathematical Point of the Exercise	Given numerical information about two different accounts, define functions that can be used to determine the cost for either account for any amount of time. Subtract the two function rules to define a function that describes cost difference between the two accounts.	Given numerical information about two different accounts, define a function which will give the cost difference between the two accounts.
Classification on TAG	Procedures Without Connections	Doing Mathematics
Reasons for Low-Level or High-Level Categorization	• Scaffolding in problems 22 and 23 reduces the level of thinking necessary to answer question 24. • "Hint" in problem 22 reduces the complexity and level of thinking required. There is little ambiguity about what needs to be done.	• No explicit pathway for how to solve the problem is provided and there are multiple solution paths. • Requires exploration of the mathematical relationship and to formulate a rule, either symbolic or verbal, that characterizes the difference between two plans in terms of the fixed charge and the per kwh charge. • Multiple representations exist (table and system of equations – verbal or symbolic).

MODIFYING COGNITIVE DEMAND OF TASKS MODULE -- ANSWER KEY

Promising Feature	Real-world Context	Real-world Context
Modifications Made to the Task		Components of the task have been eliminated to reduce the amount of scaffolding.
Common Core State Standards		F.BF.A.1, F.BF.A.1a, F.BF.A.1b, F.IF.B.4, F.IF.B.6, MP1, MP2, MP4, MP6, MP7

MODIFYING COGNITIVE DEMAND OF TASKS MODULE -- ANSWER KEY

Burning Calories while Swimming	*Original Task*	*Modification*
	37. If you swim the backstroke, you burn 9 cal/min (calories per minute). If you swim the butterfly stroke, you burn 12 cal/min. The equation $9x + 12y = 360$ models how you can burn 360 cal by swimming the backstroke for x min and the butterfly for y min. **a.** Find the solutions of the equation for $x = 0$ and $y = 0$. Explain what your solutions mean. **b.** Graph the solutions you found in part (a). Draw a line through the two points. **c.** The solutions you found in part (a) are the *y-intercept* and the *x-intercept* of the graph. Explain why these names are appropriate. **d.** Use your graph from part (b). If you swim the butterfly stroke for 10 min, how long should you swim the backstroke to burn a total of 360 calories? Davison, D., Landau, M., McCracken, L., & Thompson, L. (2001). Pre–Algebra, California Edition, Prentice Hall, 395.	**37.** If you swim the backstroke, you burn 9 cal/min (calories per minute). If you swim the butterfly stroke, you burn 12 cal/min. **a.** You want to burn 360 calories with your swimming workout. Determine a way to find all the combinations of swimming the butterfly and backstroke that meet this goal. **b.** Write and graph an equation that represents the relationship described in part **a**. **c.** Use your graph to answer the following: What happens when you only swim the backstroke? What happens when you only swim the butterfly? **d.** If you swim the butterfly stroke for 10 minutes, how long should you swim the backstroke to burn a total of 360 calories?
Mathematical Point of the Exercise	Given a context and a linear equation in two variables, determine intercepts, use them to sketch a graph, and interpret their meaning in the context. Use the graph to answer questions about the context.	Given a context, generate an equation in two variables, make a graph and use the various representations to answer questions about the context.
Classification on TAG	Procedures Without Connections	Doing Math
Reasons for Low-Level or High-Level Categorization	• Suggests a procedure to follow in parts *b*, *c*, and *d.* • Requires limited cognitive demand: the equation is given and values need only be plugged in. • Uses multiple representations, but does not make connections between them.	• Students are asked to explore complex mathematical relationships to write a rule with no specified procedure – can use table, graph, equation, etc. • Students must move flexibly between/among different representations of the problem and make connections between/among them. • Students must construct mathematical models of a real-world situation and consider their accuracy and meaning.

Promising Feature	• Number of calories to burn is arbitrary; could change that parameter. • Features multiple representations (graph, equation, real-world situation).	
Modifications Made to the Task		Rather than being given the equation that relates the two variables, students must determine this relationship. In part *d*, students may use any of their representations (equation, table, or graph) to answer the contextual question.
Common Core State Standards		A.CED.A.2, A.CED.A.3, A.REI.D.10, F.IF.B.4, F.IF.C.7, F.IF.C.7a, MP1, MP2, MP4, MP6, MP7

<table>
<tr><th>Parallel Lines Postulates</th><th>Original Task</th><th>Modification</th></tr>
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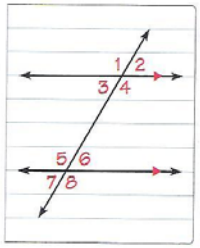
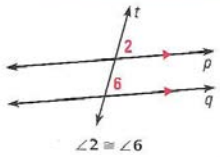
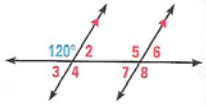
ACTIVITY EXPLORE PARALLEL LINES
Materials: lined paper, tracing paper, straightedge
STEP 1 Draw a pair of parallel lines cut by a nonperpendicular transversal on lined paper. Label the angles as shown.
STEP 2 Trace your drawing onto tracing paper.
STEP 3 Move the tracing paper to position ∠1 of the traced figure over ∠5 of the original figure. Compare the angles. Are they congruent?
STEP 4 Compare the eight angles and list all the congruent pairs. What do you notice about the special angle pairs formed by the transversal?


POSTULATE For Your Notebook
POSTULATE 15 Corresponding Angles Postulate
If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.


∠2 ≅ ∠6
EXAMPLE 1 Identify congruent angles
The measure of three of the numbered angles is 120°. Identify the angles. Explain your reasoning.


Solution
By the Corresponding Angles Postulate, $m\angle 5 = 120°$.
Using the Vertical Angles Congruence Theorem, $m\angle 4 = 120°$.
Because ∠4 and ∠8 are corresponding angles, by the Corresponding Angles Postulate, you know that $m\angle 8 = 120°$.
Larson, R., Boswell, L., Kanold, T. D., & Stiff, L. (2007). Geometry, McDougal Littell, p. 154
</td>
<td>
Activity: Explore Parallel Lines
Materials: lined paper, tracing paper, straightedge.
Step 1 Draw a pair of parallel lines cut by a nonperpendicular transversal on lined paper.
Step 2 Trace your drawing onto tracing paper.
Step 3 Number each of the eight visible angles from 1 to 8.
Step 4 Trace the angles at the top of your drawing. Use the tracings to determine which, if any, other angles are congruent to those in the tracing. What do you notice?
Step 5 Repeat Steps 1 to 4 using a transversal with a different slope.

Compare your results with those of your teammates and make conjectures about the angles formed by two parallel lines cut by a transversal.

Predict and test what will happen if 3 or more parallel lines are cut by the same transversal.
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<tr>
<td>Mathematical Point of the Exercise</td>
<td>Recognize the relationship between corresponding angles when two parallel lines are cut by a transversal.</td>
<td>Recognize the multiple relationships among angles formed when two parallel lines are cut by a transversal. Understand the postulates that result and how to recognize corresponding, alternate interior and alternate exterior angles and interior angles on the same side of a transversal in such a situation, regardless of the orientation of the parallel lines.</td>
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</table>

MODIFYING COGNITIVE DEMAND OF TASKS MODULE -- ANSWER KEY

Classification on TAG	Procedures Without Connections	Procedures With Connections
Reasons for Low-Level or High-Level Categorization	• Suggests a procedure to follow to determine congruent angles; suggests which angles will be congruent. • Suggestion to compare ∠1 to ∠5 in *Step 3* reduces the complexity and level of thinking required. • States the results for one pair of angles. • Scaffolding in Example 1 reduces the level of thinking necessary. • There is little ambiguity about what needs to be done or how to do it. • Focus is on correct answer, not understanding or meaning.	• Suggests a procedure to follow to determine congruent angles; students are expected to explore and monitor relationships among and between angles formed. • Requires students to formulate and test conjectures about angles when parallel lines are cut by a transversal, regardless of the direction of the parallel lines or the transversal. • Requires that students extend their understanding of their observations and conjectures to include three or more parallel lines. • Requires some degree of cognitive effort, since students must consider several types of relationships that exist among the angles.
Promising Feature	• Investigation is suggested. • Postulate is defined.	• Investigation into several postulates is suggested. • Making and testing conjectures is suggested. • Students must create their own representation for investigation, compare results to those of other students and extend their thinking beyond the original conjecture.
Modifications Made to the Task		• Scaffolding provided by diagram is eliminated, thus encouraging the need to talk about the positioning of assorted angles; appropriate vocabulary can be established from that talk. • Investigation is widened; generalizations, in this case postulates, are developed.
Common Core State Standards		• G.CO.A.2, G.CO.A.5, G.CO.C.9, MP1, MP4, MP6, MP7, MP8

Supporting Rigorous Mathematics Teaching and Learning

TNCore ifl

Identifying Strategies for Modifying Tasks to Increase the Cognitive Demand

Tennessee Department of Education

High School Mathematics

Rationale

There is no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics.

Lappan & Briars, 1995

By determining the cognitive demand of tasks and being cognizant of those features of tasks that contribute to their cognitive demand, teachers will be able to create opportunities for students to engage in rigorous mathematics learning.

Session Goals

Participants will:

- deepen understanding of the cognitive demand of a task;
- learn strategies for increasing the cognitive demand of a task; and
- recognize how increasing the cognitive demand of a task gives students access to the Common Core State Standards (CCSS) for Mathematical Practice.

Overview of Activities

Participants will:

- discuss and compare the cognitive demand of mathematical tasks;
- identify strategies for modifying tasks to increase their cognitive demand; and
- modify tasks to increase their cognitive demand.

Mathematical Tasks:
A Critical Starting Point for Instruction

All tasks are not created equal−***different tasks require different levels and kinds of student thinking***.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). Implementing standards-based mathematics instruction: A casebook for professional development, p. 3. New York: Teachers College Press.

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Mathematical Tasks:
A Critical Starting Point for Instruction

The level and kind of thinking in which students engage determines what they will learn.

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Human, 1997

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Mathematical Tasks: A Critical Starting Point for Instruction

If we want students to develop the capacity to think, reason, and problem-solve, then we need to start with high-level, cognitively complex tasks.

Stein & Lane, 1996

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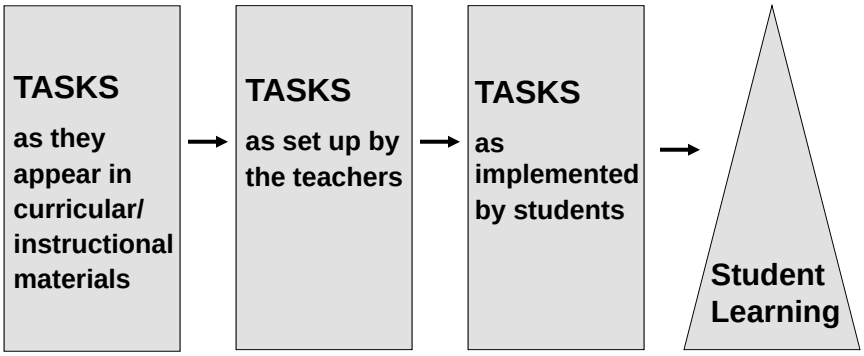
Revisiting the Levels of Cognitive Demand of Tasks

Linking to Research: The QUASAR Project

The Mathematical Tasks Framework



Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). Implementing standards-based mathematics instruction: A casebook for professional development, p. 4. New York: Teachers College Press.

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Linking to Research/Literature: The QUASAR Project

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- Low-Level Tasks
- High-Level Tasks

Linking to Research/Literature: The QUASAR Project

- Low-Level Tasks
  - Memorization
  - Procedures without Connections
- High-Level Tasks
  - Procedures with Connections
  - Doing Mathematics

Task Modification: Increasing the Cognitive Demand of Tasks

Consider…

What can you do if you want students to develop the capacity to think, reason, and problem-solve, but your textbook doesn't have many high-level, cognitively demanding tasks?

Comparing the Cognitive Demand of Tasks

Sit together in groups of three or four.

You will find the tasks on in the participant handout.

Comparing the Cognitive Demand of Tasks *(cont.)*

- Take 5 minutes to individually examine the tasks. You will find it helpful to think carefully about how students might answer each of the questions.
- Work together as a team to identify the cognitive demands of each task using the Mathematical Task Analysis Guide.

What is it that makes a high-level task "high level"?

Be prepared to present and justify your conclusions to the whole group. Be sure to identify the mathematical understandings students will have an opportunity to grapple with as they work to solve the task(s).

Comparing the Cognitive Demand of Tasks *(cont.)*

Compare the original versions of the tasks with the modified versions.

- How are the modified tasks the same and how are they different from the original?
- In what ways was the original task modified, and for what purpose?
- What is the "value added" by making the modification to the original task?
  - Which CCSS for Mathematical Practice will students use when solving each task?
  - Which CCSS for Mathematical Content are the focus of each task?

Increasing the Cognitive Demand of Mathematical Tasks

What strategies for increasing the cognitive demand of tasks may be generalized from the modifications we have just examined?

The Common Core State Standards (CCSS)

How does the modification of a task to be a high level of cognitive demand impact the opportunities to make use of the Mathematical Practice Standards?

Strategies for Modifying Textbook Tasks

Increasing the cognitive demand of tasks:

- Ask students to create real-world stories for “naked number” problems (e.g., Slope Intercepts Task).
- Include a prompt that asks students to represent the information another way (with a picture, in a table, a graph, an equation, with a context) and to write about what insights they can gain about the math concept from the new representation (e.g., Slope Task, Slope Intercepts Task).
- Solve an “algebrafied” version of the task (e.g., Write an Equation, Electricity Rates).
- Use a task “out of sequence” before students have memorized a rule or have practiced a procedure that can be routinely applied (e.g., Slope Task, Equations and Graphs Task).
- Eliminate components of the task that provide too much scaffolding (e.g., Electricity Rates, Explore Parallel Lines).
- Adapt a task so as to provide more opportunities for students to think and reason—let students figure things out for themselves (e.g., Equations and Graphs Task, Burning Calories while Swimming, Explore Parallel Lines).

Give It a Go!

Your Turn

- Form grade-level groups of no more than three people.
- Briefly discuss important NEW mathematical concepts, processes, or relationships you will want students to uncover during the lesson.
- Examine your resources for a task that can (or will) give students a chance to engage in examining those concepts, processes, or relationships.
- Then…

Your Turn *(cont.)*

- Analyze the task and consider:
  - the CCSS for Mathematical Content; and
  - the CCSS for Mathematical Practice.
- Modify the textbook task by using one or more of the Textbook Modification Strategies.
- Be prepared to share your choices and your rationale.

Comparing Tasks

What messages to students do
the differences in the tasks send?

Step Back

What have you learned about modifying tasks to increase the cognitive demand that you will use in your planning and instruction next school year?